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QUARTERLY PROGRESS REPORT NO. 6

Contract No. AF 19(122)-7, Items II & III

March 1, 1953 to May 31, 1953

Item II: Reliability Research
Item III: Coding Circuitry

Prepared by

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RELIABILITY RESEARCH AND CODING CIRCUITRY

ABSTRACT

This report summarizes the work performed under the "Reliability Research" and "Coding Circuitry" items of the contract during the period from February 15, 1953 to May 15, 1953. These items are primarily concerned with improving the overall reliability of a specific future IFF system.

Consideration is given to the implementation of a mock-up of the transmission links of the IFF system. A pulse-train correlator, optimum video and i-f filters, and redundancy and error-correcting coders are all considered for use in the system for combatting noise and pulse jam. Treatment is given to the use of Gaussian noise as the challenge and reply signals in a variation of the system considered. Speculation is made concerning the possibility of using strong enemy jam as the challenge signal, and thus provide an anti-jamming feature to protect the heretofore vulnerable ground-to-air link. Also covered is progress made in the development of a reliable shift register, and on a transistor-testing program.

a. Personnel and Administration

1. Martin W. Essigmann, Coordinator (two-fifths time Item II, one-fifth time Item III, engineer)
2. Sze-Hou Chang (half-time engineer Item II)
3. George E. Pihl (one-fifth time Item II, one-tenth time Item III, engineer)
4. John S. Rochefort, liaison man (four-fifths time Item II, one-fifth time Item III, engineer)
5. Harold L. Stubbs (half-time mathematician Item II)
6. Thomas P. Cheatham, Jr. (one-fourth time engineer Item II)
7. Walter H. Lob (full-time physicist Item II)
8. Louis J. Nardone (full-time engineer Item III)
9. Myron L. Bovarnick (full time Item II to April 13, 1953; one-fourth time Item II from April 13, 1953, engineer)
10. Jacob Wiren (full time Item II to April 20, 1953; half time Item II from April 20, 1953, engineer)
11. Anthony M. Briana (full-time engineer Item II from April 20, 1953)
12. Walter H. Goddard (two-fifths time Item II, one-fifth time Item III, technician)
13. Mary D. Reynolds (two-fifths time Item II, one-fifth time Item III, secretary)
14. Lawrence J. O'Connor (part-time assistant Item II through April 4, 1953; full-time cooperative student assistant from April 6, 1953)
15. Thomas J. White (full-time cooperative student assistant Item II from April 6, 1953)
16. Charles U. Knowles (full-time cooperative student assistant Item II to April 3, 1953; part-time assistant Item III from April 6, 1953)
17. Robert H. Lawson (full-time cooperative student assistant Item II through April 3, 1953; part-time assistant Item III from April 28, 1953)

b. Communications

1. Correspondence

Listings of all non-expendable property received for use under this contract have been sent to the Research Accountable Property Officer under the dates of February 28, 1953, March 31, 1953, and April 30, 1953.

2. Conferences

March 19, 1953. Conference at Northeastern among the entire technical staff assigned to work at Northeastern under Items II and III, and the personnel of the Communications Laboratory at AFCRC.

The purpose of this conference was to report on progress made to that date, and to discuss proposed future work.

March 23-26, 1953. M. W. Essigmann, S. H. Chang, J. S. Rochefort, and L. J. Nardone attended the 1953 National Convention of the Institute of Radio Engineers held at New York, N. Y.

March 30, 1953. J. S. Rochefort visited E. B. Staples and S. H. Reiger of AFCRC, and discussed with them work related to the transmission of signals in the presence of noise.

March 31, 1953. T. P. Cheatham, Jr. met at the AFCRC with members of the staff of the Communications Laboratory to discuss work at AFCRC related to work under Items II and III.

April 2, 1953. Visit received from Dr. Arthur Kohlenberg of M.I.T. Dr. Kohlenberg discussed problems of mutual interest with the Northeastern group.

April 11, 1953. M. W. Essigmann and T. P. Cheatham, Jr. attended the New England Radio Engineering Meeting held at Storrs, Connecticut.

April 29, 1953. L. J. Nardone visited C. R. Hurtig of M.I.T., and discussed with him the application of transistors to computer circuitry.

May 11-13, 1953. M. L. Bovarnick attended the sessions of the National Conference on Airborne Electronics held at Dayton, Ohio.

J. S. Rochefort, as liaison man between AFCRC and Northeastern has kept constant contact with AFCRC to fulfill this obligation. In addition, L. Nardone and J. Wiren have made many visits to AFCRC related to the progress of work under the recently activated Item III.

c. Statement of the Problem

Item II of the contract is concerned with research directed toward the specification of a high-reliability transmission system for use in the ground-to-air and air-to-ground links of a specified IFF system. This system is one in which the challenge as generated by the interrogator is an n-digit binary number, and an encoder at the airborne transpondor provides an r-digit reply. The reply, on reception at the responsor on the ground, is compared with a locally encoded version to determine whether or not it is correct. Primary attention is being given to the use of pulses as the signal in the two transmission links; however, for the sake of completeness some consideration is being given to other forms of signal such as noise.

The two transmission links in the system specified differ in that the correct reply is available at the terminal of the air-to-ground link, thus providing the possibility of using cross- correlation techniques in the reception of the coded reply. Less powerful means must be employed at the airplane, since the system precludes any a priori knowledge of the challenge at the transpondor. Atmospherics, enemy noise and pulse jamming, and enemy attempts at interrogation are being considered as factors which need to be combatted in obtaining high transmission reliability.

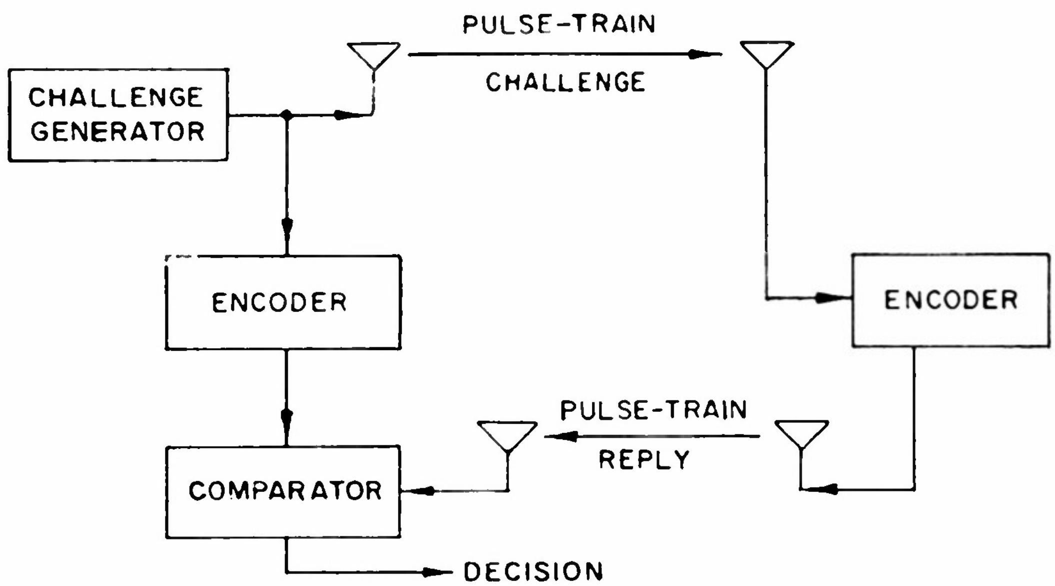
The IFF system is assumed to be secure against an enemy who can only listen; hence, the aspect of cryptographic security introduced by the presence of the two encoders in the system is not a part of the present problem.

Item III on coding circuitry as presently defined involves the design, development, construction, testing, and evaluation of a reliable transistorized shift register for eventual use in the encoder unit of the system. A supporting unit of work under this item is the transistor-testing program.

d. Methods of Attack

System Reliability

The block diagram below shows the essential features of the IFF system for which methods of improving transmission reliability are being studied. Past reports have described various methods and equipments for application in



the transmission links. During the present report period, progress has been made on the development of the various devices, with no major change in the basic philosophy insofar as the system shown is concerned. For the sake of completeness, and in order to ensure that no alternative system is being overlooked, some time has been devoted to the consideration of the use of noise as the transmission signal. It is believed appropriate to include a discussion of this work in this report.

General Considerations

Pulse system. Figure 1 of the Appendix shows a block diagram of a test set-up which can also be considered to be a mock-up of the air-to-ground transmission link. Most of the work of this period has been on the construction of equipment shown by blocks on the diagram. The section to the right of the adder represents the responsor receiver section. It includes an optimum filter designed to optimize the pulse amplitude in the presence of noise jamming, and a pulse-train correlator designed to select the correct pulse train in the presence of pulse jamming. The section to the left of the adder simulates the transmission link. Provision is made for the generation of coded r-f pulse trains (signal-generator branch), and the simulation of pulse jamming and two types of noise jamming.

With one exception this same system applies to the ground-to-air link. In this case, however, the pulse-train correlator is not applicable since storage of the challenge is not allowed at the airplane. It is expected, however, that other features for improving the operational accuracy of the system can be involved in the system for both links. These include the use of coding to match the channel capacity, and considerable work has already been done under the contract along this line. A second feature, that of the use of a directional-antenna system, has been discussed in past reports.

In the subsequent sections of this report, progress made on the development of the various equipments involved in the above system will be described, and a treatment will be given of new aspects in the redundancy-coding approach.

Noise system. The system block diagram shown above, and described for operation using pulse trains, can also be interpreted for operation using noise as the transmission signal. Justification for this approach arises from recent work in the field of information theory[1] which has shown that, subject to certain reasonable conditions, the most efficient signal from the standpoint of bandwidth-time utilization is white Gaussian noise. In fact, it has been shown[2] that a theoretical improvement of 8 db over P.C.M. is possible.

In the previous progress report it was recommended that consideration be given to a system which employs random noise as a challenge, which noise then is to be processed by a linear or non-linear network in the transpondor, whose parameters constitute the code of the sortie period.

If, for the moment, the transpondor is assumed to be linear*, the advantage of such a system lies in the fact that in the airplane no decisions as to the presence or absence of a pulse or the correctness or incorrectness of a challenge need be made, but such decisions are deferred to the responsor, where the locally encoded version of the challenge is available and cross-correlation techniques can be employed. Due to the linear nature of the transpondor, any jam that is picked up in the ground-to-air link will not be mixed with the signal but only be added to it, and will therefore be separable from the signal by the correlator in the responsor. The weakness of this system lies in the fact that the system function of such a linear transpondor can, in principle at least, be determined by the enemy, using the auto-correlation function of the input and the cross-correlation of input and output. Such compromise would enable the enemy to appear as a friend.

If, on the other hand, non-linear elements are incorporated in the transpondor in order to avoid the aforementioned difficulty, it would appear that the principle advantage of the system is lost, i.e. the pure addition of signal and jam without mixing.

* Such a system is analyzed in detail in a subsequent section of this report.

Insofar as the practicality of a system using noise as a signal is concerned, its design and development would be much more difficult than for the pulse-train system. Circuits for the modulation and detection of pulse signals are well known, and present developments in this field are in an advanced stage of refinement. Work has been begun and is advancing at various activities on the performance of pulse detectors in the presence of noise. On the other hand, less is known about the corresponding circuits required to instrument the noise system.

Filtering

The development of the matched filter for i-f pulses is still in the planning stage. The block diagram of the filter has been simplified to the form shown in Fig. 2A. The modified version of the filter permits the use of a delay line of half the original length and eliminates the need for an adding circuit.

If a voltage impulse is applied to the input of the circuit, it will appear across the inputs of the delay line and the tank circuit. Since the delay line is driven from its characteristic impedance, R_0, and open-circuited at the far end, a single reflection will occur. Consequently two positive impulses separated by the two-way delay time, δ , will appear at the input to the tank circuit. In the actual design a pentode will be inserted between the delay line and the tank circuit in order to convert the voltage impulses into current impulses. The first impulse will shock the tank circuit into oscillation and a damped cosinusoid will be produced at the output. After δ seconds have elapsed the oscillation will be stopped by the second impulse provided that the resonant frequency of the tank circuit is such that $N + 1/2$ (where N is an integer) cycles have been produced in the time δ . If the Q of the tank circuit is sufficiently high the oscillation will be essentially undamped and the envelope of the pulse will be rectangular.

A filter of this type will give a maximum output for i-f pulses which look like the backwards version of its impulse response. However, as was shown in the preceding Progress Report, the phase of the carrier is unimportant as the peak output will be within 2.7 percent of the theoretical maximum for any rectangular pulse at a carrier frequency equal to the resonant frequency of the tank circuit and a pulse width of δ seconds.

The theoretical maximum value of peak signal-power to noise-power ratio which can be obtained with a matched i-f filter is calculated in the Appendix and is shown to be an improvement of π (4.97 db) over that available from a conventional band-pass i-f filter.

A matched filter for rectangular video pulses was constructed during the previous report period. During this report period the filter has undergone a modification similar to that used in the matched i-f filter. The revised block diagram is shown in Fig. 2B. As in the case of the i-f filter, the revision permits the use of a delay line of half the original length and eliminates the need for an adding circuit.

If a voltage impulse is applied to the input of the matched video filter, it will appear across the inputs of the delay line and low-pass filter. Since the delay line is driven from its characteristic impedance, R_o, and is short-circuited at the far end, a single reflection consisting of a negative impulse will occur. Thus two impulses, the first positive and the second negative, separated by the two-way delay time δ, will appear at the input to the low-pass filter. The first impulse will give rise to an impulse response in the form of a decaying exponential. If the time constant of the low-pass filter is long in comparison with the time δ, the second impulse will cause the output to return to zero. Consequently the impulse response of the filter will be a δ-second rectangular pulse and the filter will be matched to similar video pulses.

Since the matched video filter has an impulse response which looks like a δ-second rectangular pulse, the frequency response, $A(\omega)$, of the filter should be proportional to the magnitude of the Fourier transform of the impulse response

$$A(\omega) \propto |H(\omega)|$$

$$\text{where } |H(\omega)| = \left| \int_{-\infty}^{\infty} h(t)\, \varepsilon^{-j\omega t} dt \right|$$

$$|H(\omega)| = \left| \int_0^{\delta} \varepsilon^{-j\omega t} dt \right| = \left| \frac{1 - \varepsilon^{-j\omega\delta}}{j\omega} \right| = \frac{\sin\frac{\omega\delta}{2}}{\frac{\omega\delta}{2}} .$$

In order to check the design of the matched video filter from a frequency domain standpoint, the frequency response has been determined. A close agreement has been found to exist between theoretical and experimental results and will be further discussed under e. Apparatus and Equipment.

Evaluation of the performance of the filter in the presence of noise has been undertaken. Tests have fallen under two catagories: visual observation, and measurement.

Visual testing has been undertaken with an oscilloscope. Broad-band Gaussian noise and a δ-second rectangular pulse were combined linearly in an adding circuit and applied to the input of the matched video filter. The output from the filter was observed on an unsynchronised oscilloscope. The pulse amplitude of the signal was adjusted until it was readily observed at the output. Then the oscilloscope was transferred to the input of the filter and the signal amplitude readjusted until the pulse could be discerned equally well above the input noise. The ratio of the two values of pulse amplitude was then converted to a power ratio and compared to the power ratio obtained when the same test was performed on a 0-to-$\frac{1}{\delta}$ cps low-pass filter. The resulting ratio indicated an improvement in peak-signal power to noise-power ratio of about 2 or 3. This result is the same order of magnitude as the value π which was calculated previously.*

* See Quarterly Progress Report No. 2, dated May 31, 1952.

The second method of testing employed a similar test set-up. Once more the pulse amplitude was adjusted until the pulse was readily observed at the output. Then each input to the adding circuit was separately reduced to zero and the remaining excitation measured at both input and output of the filter. The pulse amplitude was measured with a calibrated oscilloscope, and noise voltage with a wide-band rms voltmeter. Similar measurements were then made on the 0-to-$\frac{1}{2}$ cps low-pass filter and an improvement of 2.56 obtained.

The discrepancy between the above results and the computed value of π is probably largely due to the fact that the rms voltmeter was primarily designed for measurement of sinusoidal voltages. A thermocouple type of voltmeter should be used for accurate measurement of noise voltage. However, the results are close enough to the predicted values and consequently indicate that matched filters of this type can be realized with simple circuitry.

Pulse-Train Correlator

During this report period work on the pulse-train correlator has been primarily concerned with the construction and improvement of circuits in the laboratory model. Details of these circuit modifications are given under e. Apparatus and Equipment.

A theoretical evaluation of the correlator is presently being made. Special consideration is being given to the behavior of the threshold-voltage circuits in the presence of noise jamming. It presently appears that, while the threshold level should follow instantaneously the variations in received signal strength when used in the presence of pulse jamming, this may not be true in the presence of Gaussian noise jamming. Further details of the results of this evaluation will be given in a subsequent report.

Redundancy Coding

Previous progress reports have included discussions of ways in which redundancy coding can be applied to the IFF reliability problem. In the past, the challenge has been considered to consist of pulses and gaps; however, it has been pointed out* that a better signal for this application may be one in which the digits are positive and negative (or the equivalent in P.P.M.) so that the total energy in a pulse train is constant. This means that it is logical to assume the same probability, $q = 1 - p$, of error for each type of digit, and the resulting symmetry simplifies the coding problem.

To determine conditions for the optimum code, suppose that the coding period is selected so that the coded signal consists of blocks of n pulses each, where each block is to be decoded as a unit. In other words, the problem is to select, for use in transmission, a subset $A_1, \ldots, A_M$ out of the 2^n possible pulse trains of length n. The selection should be such that under the assumed conditions of noise or of random jamming, it is highly probable that the pulse train transmitted will be perturbed by the noise or jamming in such a way that it will still be decoded as that pulse train which was actually transmitted rather than some other pulse train of the subset. From this subset or catalogue of pulse trains, $A_1, \ldots, A_M$, one is chosen at random for transmission

* See Quarterly Progress Report No. 5, p 6.

at any time; thus the entropy of the transmitted pulse train is $H = \frac{\log_2 M}{n}$ bits/symbol[1].

Since every pulse has the same probability, q, of being received incorrectly, the probability, $pr(B_j/A_i)$ that pulse train A_i will be received as B_j depends only on the number of places in which A_i and B_j differ, or, representing the 2^n possible pulse trainsas vertices of an n-dimensional hypercube, the number of coordinates in which A_i and B_j differ. This number of differing coordinates will be referred to as the "distance" between A_i and B_j[3]. Two intuitively evident principles can now be stated for the design of the code: (1) A received train B_j should always be decoded, or identified, as that A_i from which its "distance" is least. (2) For a given M, the subset $A_1, \ldots, A_M$ should be selected in such a way as to maximize the minimum "distance" between pairs A_i, A_j; that is, using Hamming's[3] terminology, the points $A_1, \ldots, A_M$ should be the centers of non-overlapping spheres of as nearly equal radius as possible which among them include, as nearly as possible, all of the 2^n points.

This leads us to the type of coding considered by Fano[4] who considered the problem of determining the maximum number M of such spheres which can be found for a given n, and a given radius n - k. A quantitative statement of what can be achieved by this type of coding can be obtained following Fano's analysis and results, if specific values are assumed for the constants involved. Suppose, for example, that q = .1, p = .9. This corresponds in the case of video transmission to a signal-to-noise power ratio of 2.16 db, if the noise x is Gaussian, with mean value 0 and mean square value N; that is, if

$$f(x) = \frac{1}{\sqrt{2\pi N}} e^{-\frac{x^2}{2N}}.$$

We are assuming here a signal consisting of equally probable positive and negative pulses with amplitude $\pm\sqrt{S}$, so that

$$q = .1 = \frac{1}{\sqrt{2\pi N}} \int_{\sqrt{S}}^{\infty} e^{-\frac{x^2}{2N}} dx$$

which yields the solution $\sqrt{\frac{S}{N}} = 1.28$ or $\frac{S}{N} = 2.16$ db.

The channel capacity C corresponding to q = .1 is

$$C = \log_2 2 + .9 \log_2 .9 + .1 \log_2 .1 = .531 \text{ bits/symbol.}$$

If a 16-digit pulse-train is transmitted without coding under these conditions, the probability that it will be received correctly is $(.9)^{16} = .18$, indicating a definite need for improvement in reliability. The choice of the coding period n and of the rate of transmission of information H will determine the reliability, and, according to Shannon's theorem, if $H < C$, the probability of error can be made arbitrarily small by choosing n sufficiently large. In Fig. 3 of reference 4, the reliability, measured by the "per-unit equivocation" or fraction of transmitted information which is lost due to noise, is plotted against C for several values of n, but for fixed H = .2 bits per symbol. From the point of view of the value q = .1 assumed above, and the consequent value of C = .531, it would be of interest to plot reliability against H for this fixed C, for several values of n. A few such calculations have been made, as follows, where the measure of reliability, R, is approximately the probability that 16 bits of information will be received correctly; that is, a figure comparable to the value .18 given above for the case of no coding:

n = 2	H = .5	R = .18
n = 3	H = .67	R = .18
	H = .33	R = .64
n = 4	H = .75	R = .18
n = 6	H = .50	R = .54
n = 7	H = .57	R = .52
n = 32	H = .52	R = .79
	H = .44	R = .89
	H = .37	R = .95
	H = .31	R = .98

The first five of these codes will correct an error in at most one digit out of n, and the code for n = 7 is Hamming's single-error correcting code. The four codes for n = 32 will correct up to 4, 5, 6, and 7 errors respectively.

In the course of these calculations, an attempt was made to verify that for sufficiently large n, the reliability can be made to approach 1 if $H < C$. On the contrary, it was found that regardless of the standard of reliability adopted, the limiting value of H for large n is $H_\infty = \frac{1}{2}(1 - 2q)^2 \log_2 e$, which differs from $C = 1 + q \log_2 q + (1 - q)\log_2(1 - q)$. For the case under discussion where q = .1, H_∞ = .462, compared to C = .531. For other cases,

q = .2	H_∞ = .260	C = .278
q = .3	H_∞ = .115	C = .119
q = .4	H_∞ = .02885	C = .02905

The formula for H_∞ is obtained as follows. For very large n, the code must correct for approximately nq errors, and a variation in the reliability required will yield a negligible variation in the fraction of errors to be corrected. The number M of non-overlapping spheres of radius nq which can be formed from the 2^n points is approximately

$$M = \frac{2^n}{1 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_{nq}}$$

since the denominator is the number of points (pulse trains) which differ from any fixed one in no more than nq coordinates (digits)[3,4]. This equation can be rewritten,

$$\frac{1}{M} = \sum_{m=0}^{nq} {}^nC_m \left(\tfrac{1}{2}\right)^m \left(\tfrac{1}{2}\right)^{n-m}$$

This partial sum of a binomial expansion can be evaluated by tables of the incomplete beta-function[4,5]. However, for the present purpose where only the limiting value of $H = \frac{\log_2 M}{n}$ is of interest, it was verified, with the aid of Chapter VII of reference 6, that the normal approximation is sufficiently accurate. That is

$$\frac{1}{M} = \int_a^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt ,$$

where
$$a = \frac{\frac{n}{2} - nq}{\frac{1}{2}\sqrt{n}} = (1 - 2q)\sqrt{n} \quad .$$

To evaluate this integral for large n, integration by parts was used as on page 120 of reference 7, giving

$$\frac{1}{M} = \frac{e^{-\frac{n}{2}(1 - 2q)^2}}{(1 - 2q)\sqrt{2\pi n}} \left(1 - \frac{1}{n(1 - 2q)^2} + \ldots\right)$$

and
$$\lim_{n\to\infty} \frac{\log_2 M}{n} = \frac{1}{2}(1 - 2q)^2 \log_2 e.$$

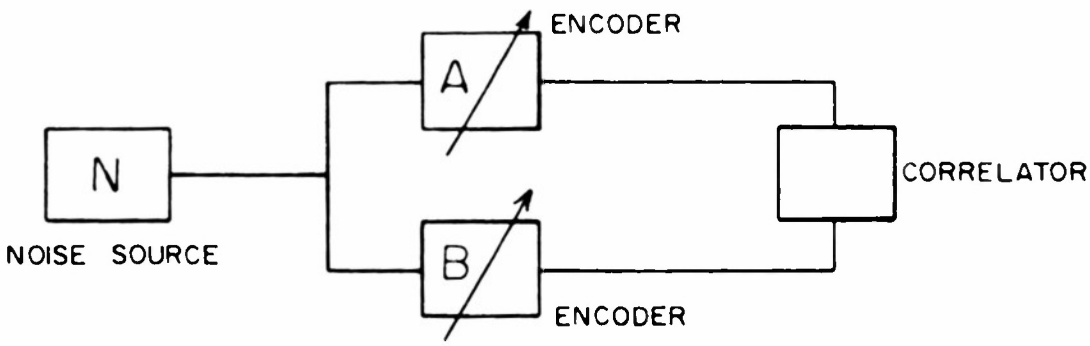
This failure of H to approach C for this type of code was noted by Fano on page 27 of reference 4 as follows: "It is clear that this limiting behavior is approached rather slowly when n is increased. For n = 67 (the largest values for which tables of [the incomplete beta-function] are available) the curve has a marked threshold behavior but still it is not close to the theoretical limit." An explanation of this apparent discrepancy has not been found. It seems evident, and has been implicitly assumed by Fano, that this error-correcting type of code as described by principles (1) and (2) above, is the optimum for the assumed noise conditions, and hence that it should obey Shannon's theorem.

If one of these codes is to be used in an IFF system, it would be desirable to use one of Hamming's "systematic" codes, such as the single-error correcting code for n = 7 which provides a convenient procedure for correcting the errors. The procedure is to add certain combinations of digits in the received message and to use the resulting sums to locate which digit or digits are in error.

In this study of redundancy coding, it has been assumed that the transmitting equipment is operating at maximum peak power but not at maximum average power. In other words, it has been assumed that additional digits can be added for redundancy without decreasing the amplitude of each pulse, and furthermore that the gain in reliability being achieved by the use of redundancy could not have been achieved by simply increasing the amplitude of each pulse. The value of redundancy in an IFF system will depend on the extent to which these assumptions apply to the transmitting equipment used in the system.

Conjecture on Using Noise as the Signal

For a fixed average power and a fixed bandwidth the maximum entropy is possessed by an ensemble which is white Gaussian noise.[1] The use of such noise as a challenging signal makes it possible to fill an assigned channel capacity more efficiently and therefore to compete favorably with man-made jamming or natural interference. In this report period some aspects of a possible scheme using Gaussian signals have been studied. The principal features are analogous to that of the main scheme under consideration except that a continuous noise source replaces the randomized discrete binary digits, as shown below:



Path A represents the transpondor in the plane to be identified and path B represents a local simulating link. The two encoders can either be linear or nonlinear, but they must be as nearly identical to each other as possible. For the present purpose of this section, only a linear encoder consisting of lumped, passive, linear circuit elements will be analyzed. The correlator under consideration is a simple analog computer using the polar portrayal of the joint distribution of the signals from the two paths. Such a correlator is discussed in detail in a recent report* of an unclassified project. It is deemed especially applicable to Gaussian signals.

Correlator of Gaussian Signals. When two Gaussian signals of zero mean and equal variances are applied to the x- and y-axes of an oscilloscope the distribution follows a pattern of elliptical symmetry. It is found that the

* Quarterly Progress Report No. 16, Contract No. AF 19(122)-7 Item 1. Visual Message Presentation, Feb. 8, 1953 to May 8, 1953, Northeastern University, pp 10-16.

distribution of intensity of illumination with respect to the polar angle θ is given by the equation:

$$p(\theta) = \frac{\sqrt{1 - r^2}}{2\pi(1 - r \sin 2\theta)}$$

where $p(\theta)$ is the distribution density and r is the normalized correlation coefficient of the two signals. It is to be noted that when $r = 1$ (or -1), the pattern becomes a straight line making an angle 45° (or 135°) with the x-axis. When $r = 0$, the pattern has a circular symmetry and the polar distribution is uniform. For intermediate values of r, maximum density M is observed at $\theta = 45^\circ$ and minimum density m at $\theta = 135^\circ$, or vice versa in case r is negative. These two densities are sufficient to determine the particular value of r, by the following formula,

$$r = \frac{M - m}{M + m} \quad .$$

An alternative method is to determine the total illumination (call it P_1) of the first quadrant and that of the second quadrant (call it P_2). Because of symmetry the illumination of the third and fourth quadrants are also equal to P_1 and P_2 respectively. Then the correlation coefficient can be computed from the formula

$$r = \sin^{-1} \frac{\pi(P_1 - P_2)}{2(P_1 + P_2)} \quad .$$

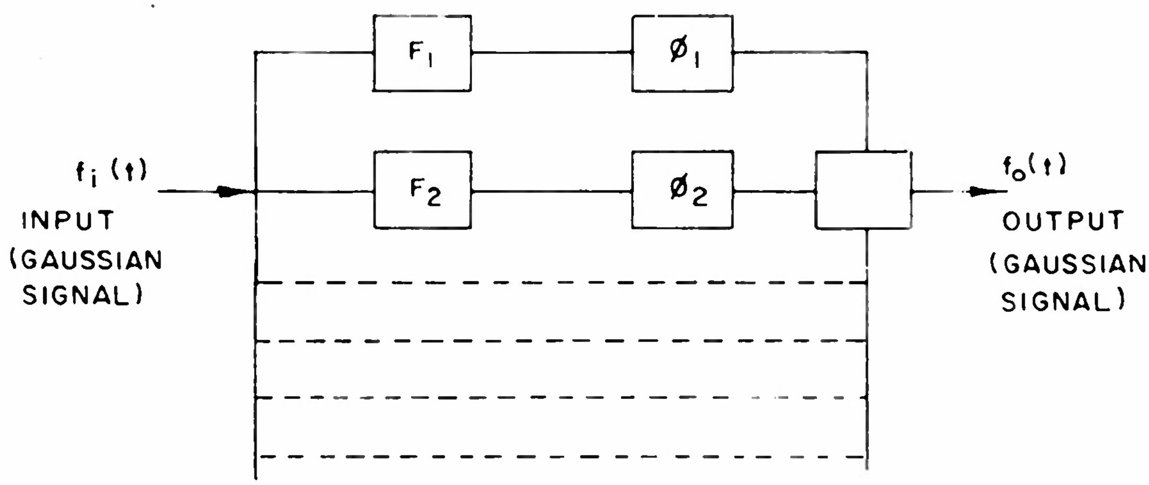
P_1 and P_2 may also be determined by infinitely limiting the x and y signals and measuring the intensity of the two beam spots in the first and second quadrants respectively.

One main advantage of this correlator is that no high-speed function multiplier is needed, and the calculations involved are simple addition and division to be performed at low speed. It is hoped that simple photoelectric and mechanical devices can be made to perform this task.

Possible Networks for Linear Encoders. The purpose of the encoder is to distort and scramble the incoming signal in a prearranged way so that its output, although still Gaussian in character, will have an entirely different waveform from that of the input. The process of distortion and scrambling, together with the necessary delay and attenuation due to radio propagation, are simulated in a local link, so that the cross-correlation of the two outputs exceeds a threshold value.

Many methods are available in the design of linear networks of arbitrary characteristics. It is preferable, however, that the network be composed of simple units, connected in parallel and/or in cascade. It is also preferable that the number of isolating devices such as vacuum tubes or transistors should be reduced to a minimum. The elements of the component units may be adjusted separately to produce necessary codes for security.

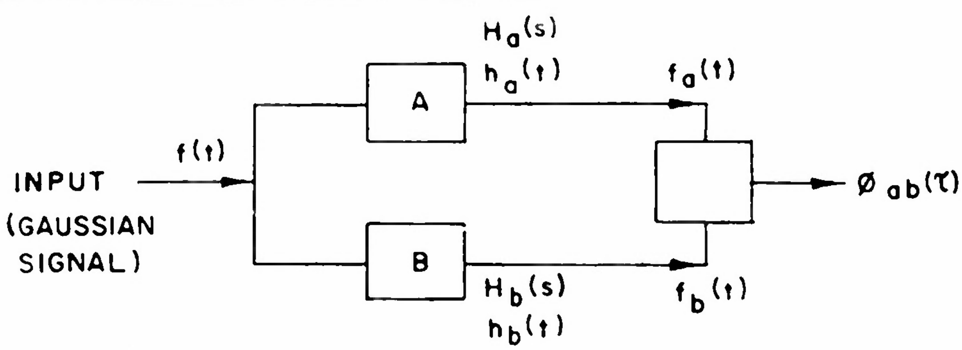
One simple scheme to produce systematic codes of various degrees of complexity is to divide the spectrum into a number of complementary bands, each band to be phase-distorted by several stages of all-pass networks. The outputs of these bands are combined again to form the output of the encoder. A block diagram of such an encoder is shown as follows:



Designs of complementary filters* of various classes and all-pass phase-distorting networksare available elsewhere and need not be discussed here.

The overall performance of the linear encoder can be described by the transfer characteristic $H(s)$ or its equivalent, the unit impulse response, $h(t)$. It is theoretically[6] possible that the enemy can, by cross-correlating the input and output signals of the encoder installed in a friendly plane, determine the transfer characteristic $H(s)$ or $h(t)$; he may thereby design an approximating network and pose as a friendly plane. This, indeed, is a fundamental limitation of the linear encoder. A further analysis expresses this limitation in a form which enables one to evaluate the situation more critically.

<u>Correlation of Outputs of Two Encoders.</u>



* See, for example, Quarterly Progress Report No. 14, Contract No. AF19(122)-7 Item I, Visual Message Presentation, Aug. 8, 1952 to Nov. 8, 1952,Northeastern University, pp 31-40.

The transfer characteristics of the encoders of the two paths are expressed as $H_a(s)$ and $H_b(s)$, or alternatively as the unit impulse responses $h_a(t)$ and $h_b(t)$. Path B is the local link while path A may be the plane of a friend or foe. The problem is to find the cross-correlation $\phi_{ab}(\tau)$ between $f_a(t)$ and $f_b(t)$ as a function of $h_a(t)$, $h_b(t)$ and the statistical property of $f(t)$.

The outputs of the two paths may be expressed as the convolution integrals:

$$f_a(t) = \int_{-\infty}^{\infty} h_a(\xi)\, f(t - \xi)\, d\xi$$

$$f_b(t) = \int_{-\infty}^{\infty} h_b(\eta)\, f(t - \eta)\, d\eta \ .$$

The cross-correlation function of $f_a(t)$ and $f_b(t)$ is, therefore,

$$\phi_{ab}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f_a(t) f_b(t + \tau)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt \int_{-\infty}^{\infty} h_a(\xi) f(t - \xi)\, d\xi \int_{-\infty}^{\infty} h_b(\eta) f(t + \tau - \eta)\, d\eta$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_a(\xi) h_b(\eta)\, d\xi\, d\eta \cdot \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(t - \xi) f(t + \tau - \eta)\, dt$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_a(\xi) h_b(\eta)\, d\xi\, d\eta\ \phi_{11}(\tau - \eta + \xi)\ ,$$

This expresses the general relation between $\phi_{ab}(\tau)$ and the characteristics of the two paths and the autocorrelation function of the input. When $\tau = 0$,

$$\phi_{ab}(0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_a(\xi) h_b(\eta)\, \phi_{11}(\xi - \eta)\, d\xi\, d\eta\ .$$

In case $f(t)$ belongs to a white noise ensemble, then

$$\phi_{11}(\xi - \eta) = \delta(\xi - \eta)$$

which is a delta function. This means that

$$\int_{-\infty}^{\infty} h_a(\xi) h_b(\eta)\, \phi_{11}(\xi - \eta)\, d\eta = \int_{-\infty}^{\infty} h_a(\xi) h_b(\eta)\, \delta(\xi - \eta)\, d\eta$$

$$= h_a(\xi) h_b(\xi)$$

and hence,

$$\phi_{ab}(0) = \int_{-\infty}^{\infty} h_a(\xi) h_b(\xi) d\xi .$$

By changing the variable of integration ξ into t and by using Parseval's theorem,

$$\phi_{ab}(0) = \int_{-\infty}^{\infty} h_a(t) h_b(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_a(\omega) H_b^*(\omega) d\omega .$$

In the case of a friendly plane, $h_a(t)$ should be almost identical to $h_b(t)$ and the value of $\phi_{ab}(0)$ is comparatively high. The success of an enemy plane in posing as a friendly plane will depend upon how well he approximates $h_b(t)$ in the design of $h_a'(t)$, based upon the measurement of $h_a(t)$ of a friendly plane. This last measurement is assumed possible through the cross-correlation of the input and output signals of a friendly plane.

A New Approach to the Ground-to-Air Jamming Problem

In order to discuss the relative success of an IFF system against jamming, it is convenient to view the probability of an error being introduced by jamming action as separable into two parts, defined below as α and β errors:

α = probability of a friend being identified as an enemy
β = probability of an enemy being identified as a friend.

In the present system, β is of a relatively low order of magnitude, being $O(2^{-16})$ if a 16-digit coding system is used, and is even further reduced if post-detection integration of multiple replies is utilized. Therefore, serious consideration need now be given to errors of the α-type. In the air-to-ground link, it is expected that with the optimum use of cross-correlation for the particular jam signal being used, the α-type error can be held to a safe operational level.

However, the same problem in the ground-to-air link has continued to grow in proportion to our ability to minimize the jamming problem on the ground. This has resulted from the realization that our anti-jamming success on the ground has been very dependent on the use of cross-correlation detection - a technique that in any simple form has been impossible in the air. It is from this realization that some theoretical consideration has been given during this report period to systems involving only a linear transformation in the air encoder. The advantage would be that the ground correlator would carry the full burden of making the decision, with the important advantage of cross-correlation techniques. The disadvantage as previously pointed out is the potentially large increase in the β-type error. Theoretically a linear transformation is easily analysed by a cross-correlation of the input and output of the system, and security from a large β-error would lie solely in our ability to build a linear transformer of sufficient complexity to defy the enemy's ability to synthesize it in the "security-time-interval".

An alternate solution has presented itself and is now under further consideration and detail study. The principle involved is quite simple. Consider the presently conceived IFF system in its simplest form, consisting, let us say, of two encoders, A and B. To operate the system, the two encoders are simultaneously challenged by a random 16 binary-digit number derived from a random interrogator located on the ground with one of the encoders. If the replies from the encoders are the same (as determined by the comparator) then A, the ground encoder, knows that B is a friend. Let us now suppose that a jamming signal is sent out from point C so that encoder B now receives the sum of two challenges. Without cross-correlation techniques available, B is unable to distinguish between the challenge from A and the jamming signal, provided they are sufficiently alike so that ordinary linear filtering can no longer differentiate between them. Encoder B then responds to the sum and is identified by A as an enemy, and the error is of the α type. However, it is pointed out that the error in this case is due to our own stubbornness in insisting that the two encoders respond to the friendly challenge. Note that if the enemy is able to jam the system, it is only because his jamming signal is so like the friendly challenge that we cannot differentiate between them, i.e., the jamming signal is just as good a challenge as the friendly signal. Why not then let the enemy be the challenger and turn the friendly challenge off! B is identified as a friend of A if both answer <u>any</u> and all allowable questions in the same manner.

It is suggested, therefore, in order to make the entire system compatible and capable of operation with even greater anti-jamming security than the presently conceived ground system, that attention be now given to the instrumentation of automatic controls for "self" and "enemy" interrogation. The fact that the optimum instrumentation and operation breaks down into a number of sub-problems involving integration with certain radar information is recognised but will not be further elaborated here at this time.

Further development and evaluation of the possibilities suggested by this approach will be combined with the theoretical evaluation of the effect of random-noise jamming on the ground correlator as part of work of the next report period.

Coding Circuitry

Shift Register

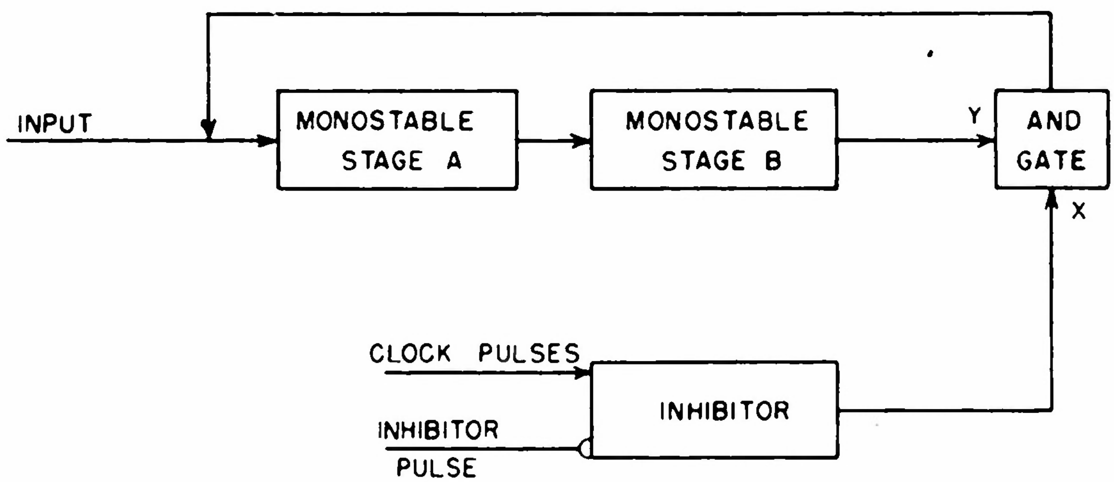
Work under Item III is concentrated on the design, construction, development, and evaluation of a reliable shift register for use in the encoder components of the overall IFF system. The fact that several of these are required, and some are involved in airborne equipment, makes it necessary that weight be reduced to a minimum. Hence, it is desirable to consider the use of transistors instead of vacuum tubes for such devices.

The specifications set up in the previous report of this contract will be continued to be used. The specifications are as follows: The input to the register will consist of 16-digit binary numbers, each pulse having a width of 0.1 to 0.3 μs, and the pulses spaced by 1 μs. The register is to be arranged for either serial or parallel read-in and either serial or parallel read-out. The loading at present is unspecified; however, it will be necessary, as the circuit develops, to investigate this aspect since the registers in practice will be connected to devices such as matrix switches.

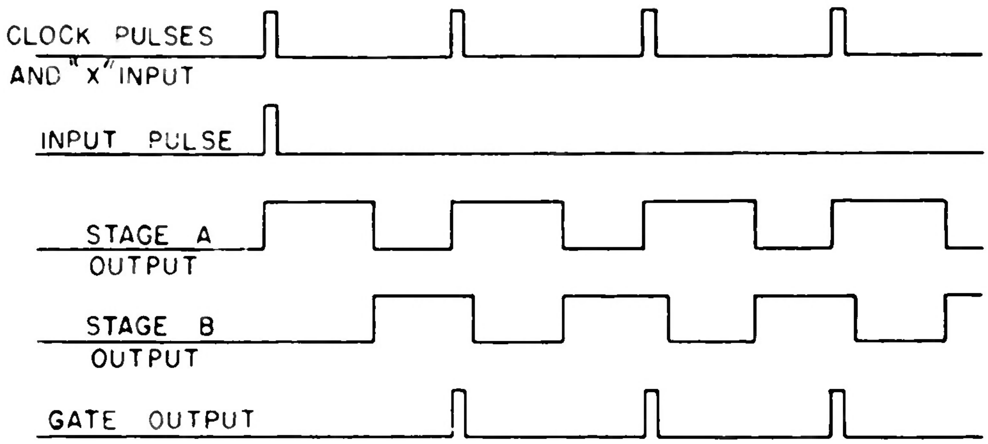
The first system used was that suggested by AFCRC in which a dynamic delay-line storage-cell is used for each digit register. Several of these cells were built and made to satisfactorily operate individually. A slight improvement in stability was obtained by introducing a small positive bias voltage (about 0.8 volts) in the base circuit of the amplifier included in the storage-cell unit. Certain transistors used (Transistor Products Type 2C and 2D), which tended to oscillate in the amplifier circuit, were made stable by the addition of this bias voltage.

However, when several of these digit registers were cascaded through amplifiers and shift gates in building up a shift register, satisfactory operation could not be obtained. Time delays, introduced through amplifiers and other yet undetermined sources, prevented the coincidence of pulses at the inputs of the shift gates resulting in a stored "one" being lost as a "zero". Means of locating and eliminating these time delays are being investigated.

The appearance of these undesirable time delays led to the consideration of a second type dynamic storage-cell using two monostable transistor circuits. This type of storage-cell, including an inhibitor circuit and an "and" gate, is shown in block diagram form below.



Clock pulses applied to the input of the inhibitor also appear at the "X" input of the gate circuit, provided that no inhibitor pulse is present. A pulse at the input of the storage cell initiates monostable stage A. The return of stage A initiates stage B. The combined delay of stages A and B must be greater than the time interval between clock pulses but less than two times the time interval between clock pulses. In this manner, the gate circuit is made operative by having pulses at the "X" and "Y" inputs simultaneously. The output pulse from the gate circuit again initiates the stage A causing the continuous repetition of the above cycle. The timing diagram for this type storage cell is shown on the next page.



A storage cell of this type has been made to operate continuously for several hours at a time. Means of improving the cell for greater reliability and stability and for optimum circuit values are at present being investigated. As soon as this phase of the work is completed, it is intended that an eight-stage storage register will be constructed.

<u>Transistor Testing</u>

The transistor-testing program has been continued during this report period to provide design data upon which to base the work on transistor circuitry carried on at Northeastern and AFCRC.

Transistors are being measured with respect to the usual small-signal parameters; namely, alpha (α) and the standard four-terminal network resistances r_{11}, r_{12}, r_{21} and r_{22} for the point-contact type, and the corresponding quantities for the junction type.

The frequency-response characteristics of the point-contact transistors are determined by measuring α as a function of frequency, and defining the frequency where α is 3 db below its value at 5 kc as the cut-off frequency.

Large-signal testing provides a method for measuring the rise time, fall time and hole-storage (or turn-off) time. The measurement of hole-storage time has been discontinued at present due to apparent changes produced in the small signal parameters by this test. A value obtained from previous measurements has been included in the results to provide a representative figure for this parameter. Table I of the appendix gives specific details of the transistors tested to date. The equipment used in making these tests has been described in previous reports*.

* See Quarterly Progress Reports No. 4, pp 16-17, 21; and No. 5, pp 16-17.

e. Apparatus and Equipment

Filtering

Construction of the matched i-f filter has not started as yet since the filter is still in the design stage. RG 62/U cable has been procured for use as a delay element. Wide-band deflection amplifiers have been purchased in order that waveforms may be observed on available oscilloscopes. It is expected that construction will commence during the next report period.

The schematic of the adding circuit and matched video filter is shown in Fig. 3. Two inputs are provided: one for signal pulses and the other for noise. The first tube and its associated circuitry comprise the adding circuit, while the second tube serves as delay-line driver.

The frequency response of the circuit was measured between the signal input and output terminals and is plotted in Fig. 4. The crosses indicate the experimental data while the smooth curve was plotted from the expression

$$\frac{\sin\left(\frac{\omega\delta}{2}\right)}{\left(\frac{\omega\delta}{2}\right)} \qquad \text{where} \qquad \delta = 0.645\mu s.$$

Although the delay line was cut to give 0.6 µs two-way delay time, there is evidently some discrepancy between the time and frequency measurements as it was found that the theoretical curve for $\delta = 0.645$ µs was the best fit to the experimental data. The close agreement between the experimental data and theoretical curve serves to illustrate the accuracy which can be achieved with the design incorporated in the matched filter.

Since the procedure for testing the performance of the filter in the presence of noise was described in detail under d. Methods of Attack, it will not be restated here. A General Radio Type No. 1390-A random-noise generator was used on the 0 to 5-mc range as a noise source. A Ballantine Model 304 voltmeter was used for noise measurements. Since the Ballantine meter is not a thermocouple-type voltmeter, the noise-power measurements cannot be considered to be very accurate. Consequently the discrepancy between the measured improvement ratio of 2.56 and the theoretical ratio of π, is probably largely due to the Ballantine meter.

The matched video filter was constructed in order to test design procedure and experimentally verify theoretical calculations. As such the filter has served its purpose and consequently the more difficult task of constructing a matched i-f filter will now be undertaken.

Pulse-Train Correlator

In this report period the correlator model was completed, and many of the troubles that were first experienced with it have been cleared up. At the present time it can be said that all parts of it behave qualitatively correctly, but that more detail work is required to obtain the accuracy necessary for successful over-all operation.

The following changes were found to be necessary and have been incorporated to date:

(1) Careful decoupling of all in-line amplifiers were required to remove oscillations.

(2) The pulse transformer in the threshold device was found to produce excessive feed-through due to capacitive coupling between primary and secondary. The threshold circuit was therefore redesigned so as to avoid the need of a transformer. By changing the adding bus connections from the +m and -k switch terminals to the -m and +k ones, the adding bus pulse was given negative polarity. Adding the positive threshold voltage to this will yield a negative result if the adding bus signal exceeds the threshold voltage, and a positive result if the adding bus signal is of smaller magnitude than the threshold voltage. Therefore the actual threshold operation can now be performed by a half-wave rectifier that passes only negative signals.

(3) The redesign of the threshold circuit required that the gains of the "A" and "B" d-c amplifiers be increased. Consideration of the revised circuits (Figs. 5 and 6) shows that the effective parameters A and B, appearing in the threshold voltage formula $V_T = AJ - BS$, are one half of the gains of the respective amplifiers. These gains were raised to about ten and twenty respectively.

(4) It was found that great care had to be taken in the cutting, connecting, and locating of the delay cable. A technique had to be developed for grounding every strand of the shield without overheating the cable to the point of collapsing its plastic foundation. Attention had to be paid to having the shield stop at the same axial position as the coil of the inside conductor, in order to avoid inductive or capacitive loading of the line. It also was found that the shield was not completely effective, so that cross-talk would occur between cables that are spaced too closely.

(5) Close control of phase-shifts in the d-c amplifiers was found necessary to ensure accurate coincidence of the adding-bus signals. This work is still in progress.

Pulse Modulator

A pulse modulator has been developed for use with the optimum filter operating at the intermediate frequency of the system. The wiring diagram is shown as Fig. 7.

In this circuit the first 6AQ5 is employed as a pulse amplifier, accepting a 10-volt positive pulse and delivering a negative pulse of sufficient amplitude to cause the 1N34 crystal diode to open. This removes the shunting effect of the low-impedance crystal from the tank circuit of the 6C4 used as a Q-multiplier, and allows it to be excited by inductive coupling to the 6AK5 oscillator during the pulse interval. The output of the

Q-multiplier is amplified by the second 6AQ5 to provide a 10-volt pulsed-carrier output. The unit is operable over the range from 25 to 60 mc and the shape of the pulse envelope may be controlled by the 25-K feedback resistor of the Q-multiplier.

Mock-up System

The test set-up which will be used to simulate the air-to-ground link of the IFF system under consideration is shown in Fig. 1. Much of the equipment indicated in the block diagram is available or under construction.

The signal generator and jamming-pulse generator were constructed some time ago and are discussed in earlier reports. Construction of the pulse modulator was completed during this report period and, if testing indicates that the device is satisfactory, a duplicate unit can be constructed for use as a jamming-pulse modulator. A General Radio Type 1390-A random-noise generator is available for use as a 0 - 5 mc low-frequency noise source. A Kay Electric Mega-Node Noise-Diode is available for use as a 0 - 220 mc high-frequency noise source. The pulse-train correlator has been completed and is undergoing trouble-shooting at the present time. Construction of the optimum matched filter for i-f pulses will commence during the next report period.

As yet little thought has been given to the low-frequency-noise modulator, the adding circuit, or detector. However, these units are fairly conventional and little difficulty is expected with their design. Design and construction will probably commence during the latter part of the next report period.

The improvement in signal-to-noise ratio which can be achieved with the system shown in Fig. 1, can be experimentally determined by merely replacing the optimum filter with an i-f strip, and the correlator with a suitable indicator such as an oscilloscope.

Coding Circuitry

The construction and operational checking of the clock-pulse generator and the signal generator described in the previous report have been completed.

Several dynamic delay-line storage-cells have been built and operated. The circuit is essentially the same as that suggested by AFCRC.

A schematic diagram of the storage cell using two monostable transistor circuits is shown in Fig. 8. Stage A is a monostable circuit using a coil in the base circuit as a means of generating a time delay. The coil and the germanium diode in the collector circuit are used to generate a negative pulse at the end of the delay. The negative pulse base-triggers stage B which in turn generates a delay by means of the capacitor in the emitter circuit. The inhibitor circuit is the same as that suggested by AFCRC. The gate circuit is a conventional diode "and" circuit.

g. Conclusions and Recommendations

As a result of the work of this report period it is concluded that:

1. The matched video filter is satisfactory since test results demonstrate that good accuracy can be obtained with simple circuitry.

2. The techniques suggested in this report for constructing a matched filter for application in the i-f section of a pulse receiver appear to be practical. It is hoped that results comparable to those obtained with the matched video filter will be achieved.

3. The pulse-train correlator shows promise of success, in that all component circuits operate individually, and some tentative results of overall performance have been obtained. The effects of mutual loading and of phase shifts need to be studied in greater detail.

4. When the signal-to-noise ratio is not much greater than one, considerable improvement in reliability can be obtained by error-correcting redundancy coding, involving an increase in the length of a pulse train by a factor of 2 or 3.

5. The use of a continuous Gaussian noise as the signal will help to reduce errors due to jamming or natural interference.

6. The correlator using the polar distribution of the joint portrayal of the signal outputs of the two paths is simple and especially suitable for Gaussian signals.

7. The linear encoder for Gaussian signals has the advantages of being simple in design and adjustment, but its limitation (i.e. the ease with which its characteristics can be determined and duplicated by an unfriendly plane) requires critical analysis.

It is recommended that the system discussed in the above three parts be studied for use as a possible auxiliary channel, with the main burden of avoiding errors due to identifying foes as friends being placed on the digital system or non-linear encoding.

8. The use of jamming signals to perform the challenge function, as described herein, provides a novel and promising means for improving system reliability when the jam is large.

9. The signal generator and clock-pulse generator for storage-cell testing operate satisfactorily for their intended usage.

10. Work on the delay-line storage cell should emphasize the elimination of undesired time delays.

11. The storage cell using two monostable transistor circuits provides a promising means of instrumenting a reliable shift register.

12. The transistor testing program as presently conducted provides pertinent information for the design and evaluation of transistor circuits.

h. Future Work

In view of the work of this and previous report periods, and the above conclusions and recommendations, it is intended that future work include:

1. The construction and testing of a matched filter for application in the i-f section of a pulse receiver.

2. The construction and testing of the units of the mock-up system not as yet worked on, and the eventual assembly and use of the entire system.

3. The remodeling of the pulse-train correlator into its final form, and the testing of the unit. This will include determining its response to any or all of the following inputs:

(i) The "correct" signal,
(ii) Any "incorrect" signal (i.e. pulse jam),
(iii) Noise jam.

4. Further investigation of the discrepancy between the channel capacity and the limiting value for rate of transmission with an error-correcting code.

5. The explicit definition of a particular error-correcting code considered most suitable for the requirements of an IFF system.

6. Experimental study of the correlator and linear encoder for noise signals.

7. Theoretical study of non-linear encoders for noise signals.

8. Further investigation of the practicability of making the jamming signal perform the challenge function in an IFF system.

9. The continuation of the study and design of a storage cell suitable for use as the memory element in a shift register. All immediate work under Item III will be directed toward the development of a reliable transistorized shift register.

10. The continuation of the transistor testing program in its present form to provide data for use in transistor-circuit design and evaluation.

APPENDIX

a. Curves, Drawings and Tables

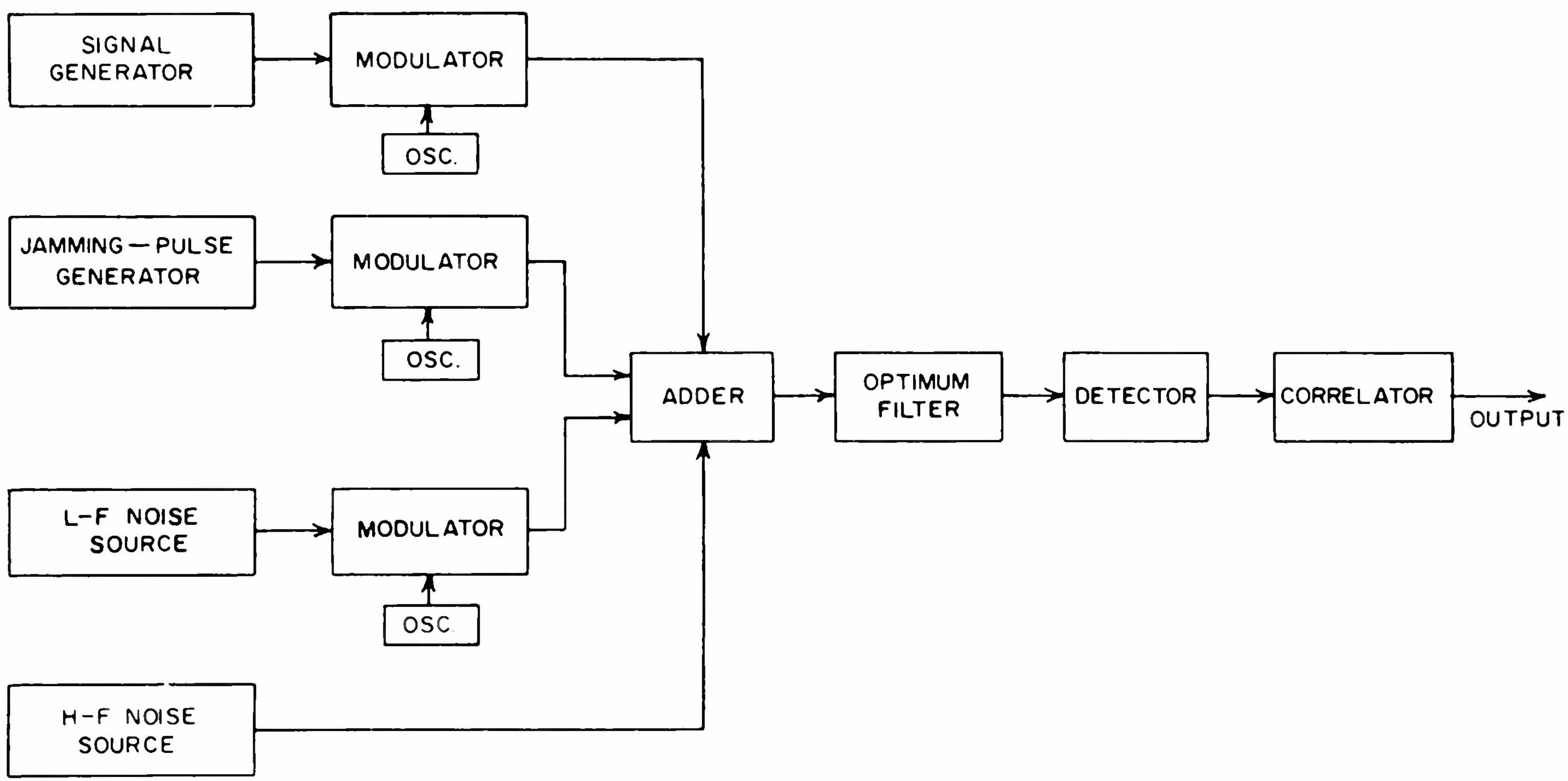
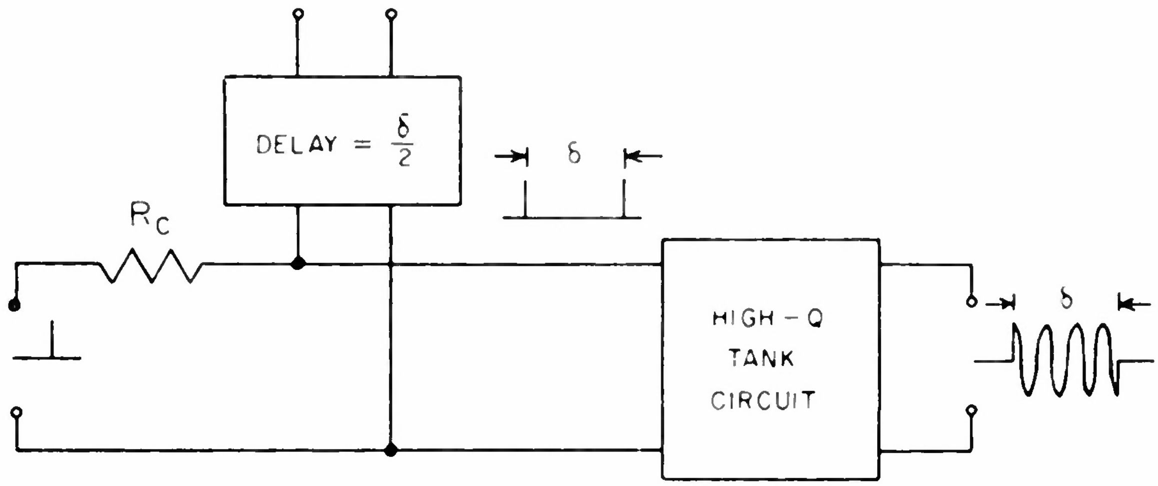
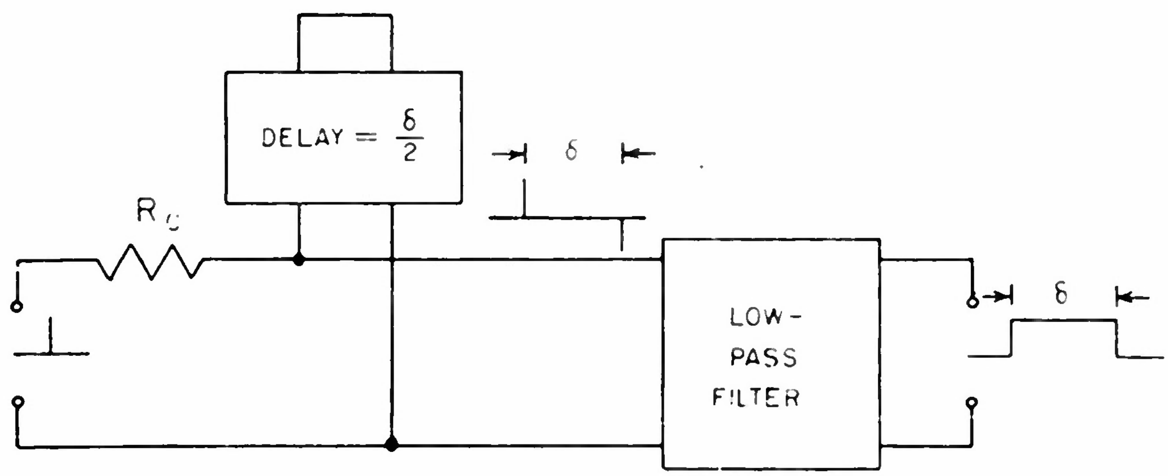


FIG. I. BLOCK DIAGRAM OF PROPOSED TEST SET-UP.



A. MATCHED FILTER FOR I-F SECTION.



B. MATCHED FILTER FOR VIDEO PULSES.

FIG. 2. MATCHED FILTERS.

TABLE I

Point-Contact Transistor Parameters, Mean Value With Standard Deviation

Manufacturer and type	No. tested	r_{11} ohms	r_{12} ohms	r_{21} kilohms	r_{22} kilohms	α	Cut-off freq., mc	Rise time, µs	Fall time, µs	Hole storage time, µs
Western Electric Type 1698	576	331±99	163±63	92±65	24±5	2.22±.49	2.02±1.26	.32±.23	.36±.23	3.41±1.22
Western Electric Type 1689	145	345±108	161±63	48±24	22±10	2.19±.40	1.73±1.04			
Western Electric Type 1768	284	244±111	84±27	32±10	11±2	2.78±.47	.511±.259	.71±.27	1.06±.35	4.98±1.88
Western Electric Type 1729	10	227	101	24	10	2.65	3.28	.09	.12	
RCA Type TA165X	91	274±68	120±42	52±21	25±8	2.04±.51	2.69±1.29	.13±.08	.17±.11	.35±.19
Transistor Products Type 2A	6	297	135	53	23	2.30	.711	.70	.94	3.09
Transistor Products Type 2C	51	380±139	235±110	50±27	18±8	2.58±.36	3.52±1.61	.13±.06	.17±.14	
Transistor Products Type 2D	124	396±185	242±96	49±25	19±10	2.28±.72	2.79±.60	.17±.08	.19±.11	
Transistor Products Type X10	207	340±138	205±86	38±17	16±7	2.30±.39	3.35±1.41	.11±.09	.11±.09	
Raytheon Type CK716	41						.872±1.18	.76±.36	1.12±.48	.69±.79
General Electric Type G11	46	553±83	303±91	60±24	25±8	2.39±.33	3.70±1.11	.13±.04	.12±.03	

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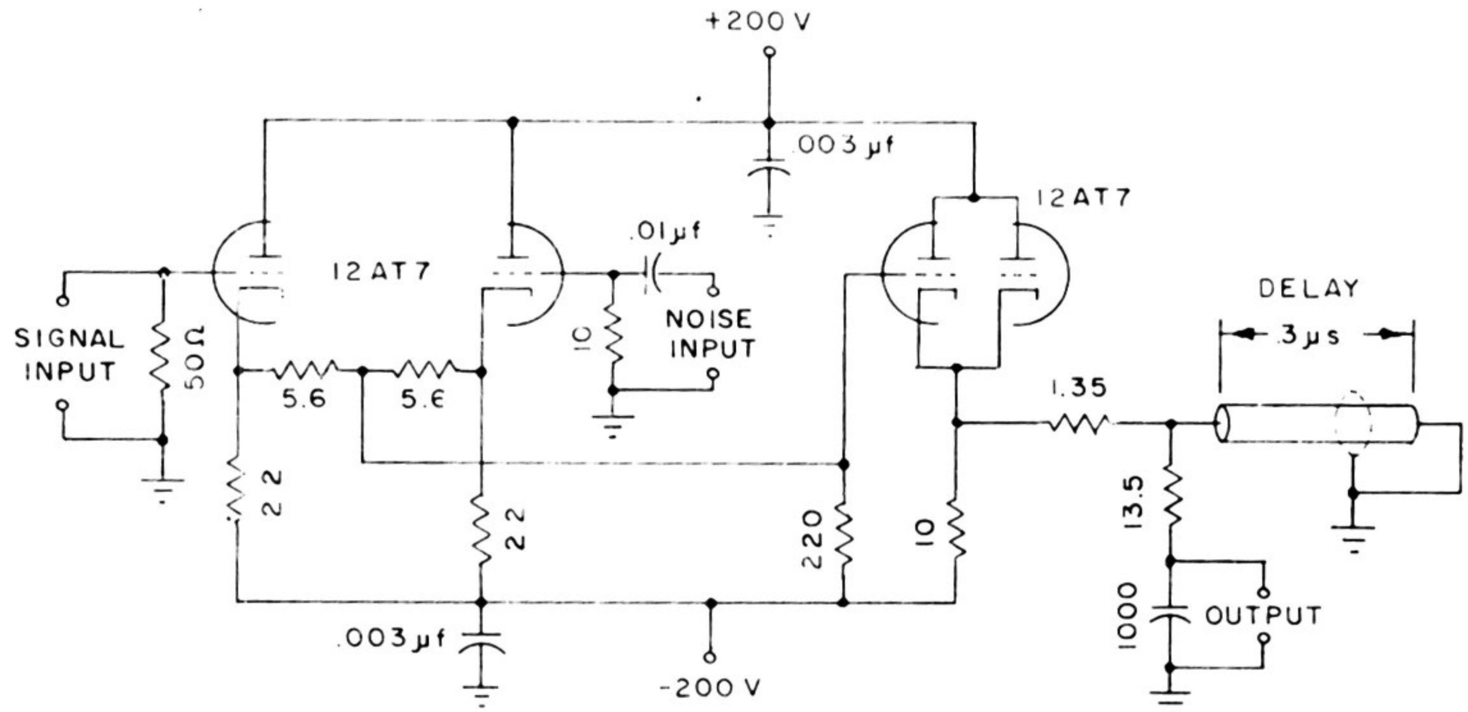


NOTE: ALL RESISTANCE VALUES IN KΩ AND ALL CAPACITANCE VALUES IN μμf UNLESS OTHERWISE SPECIFIED.

FIG. 3. SCHEMATIC OF ADDING CIRCUIT AND MATCHED VIDEO FILTER.

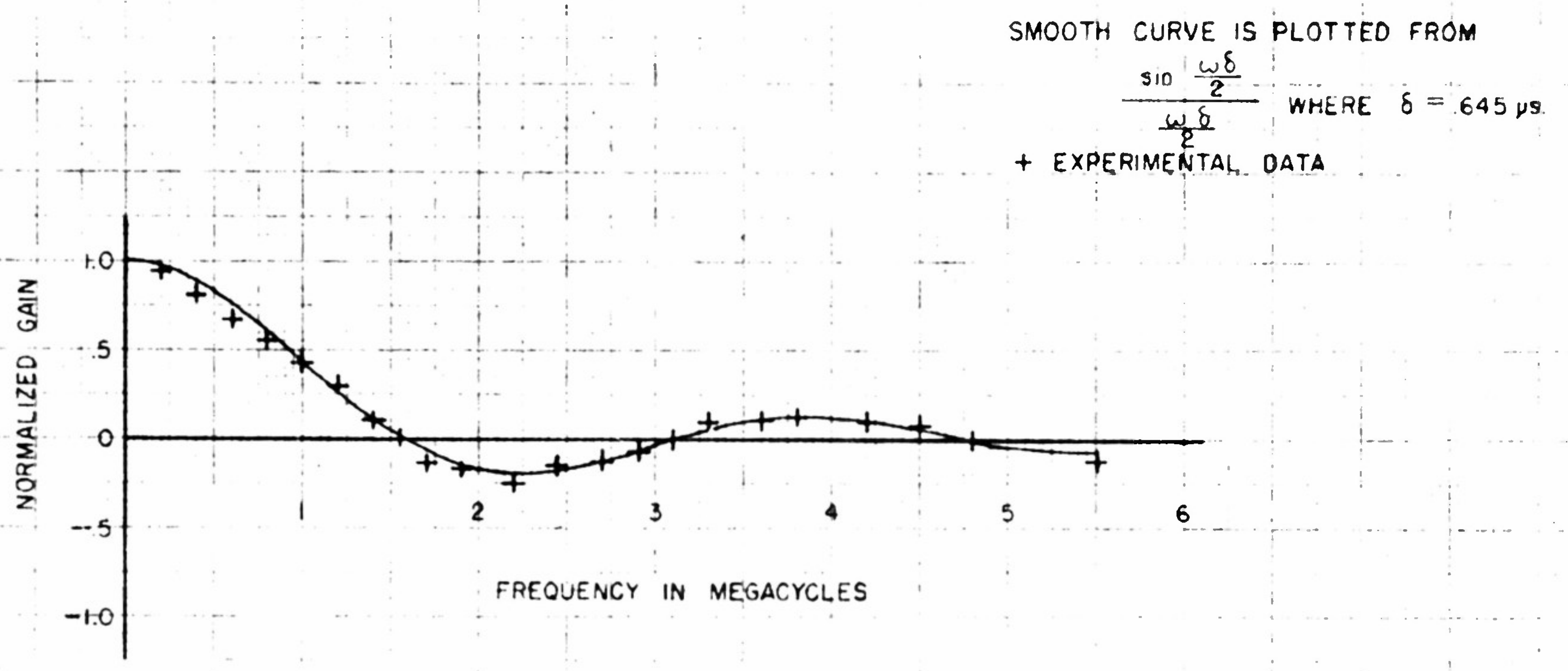
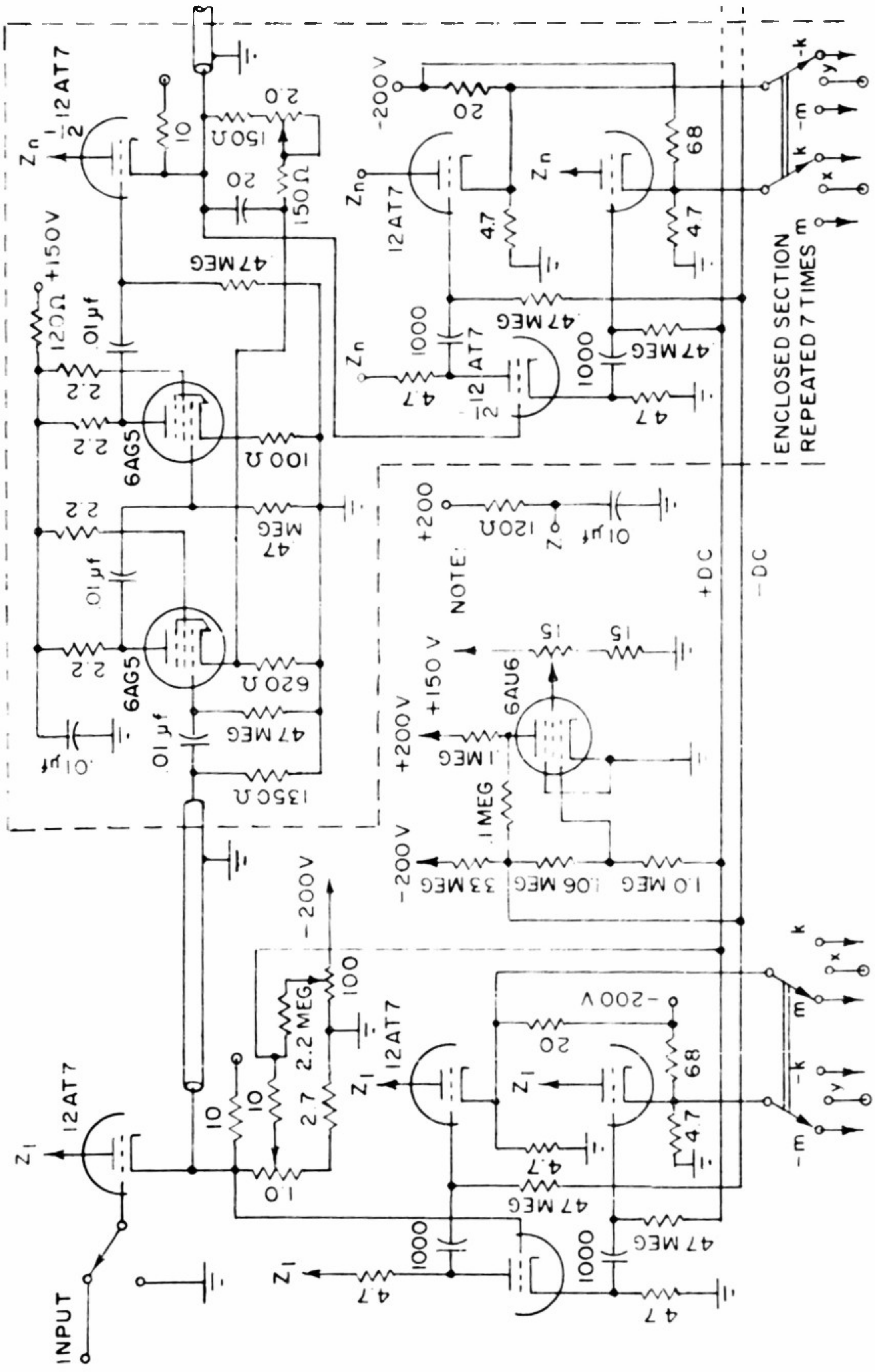
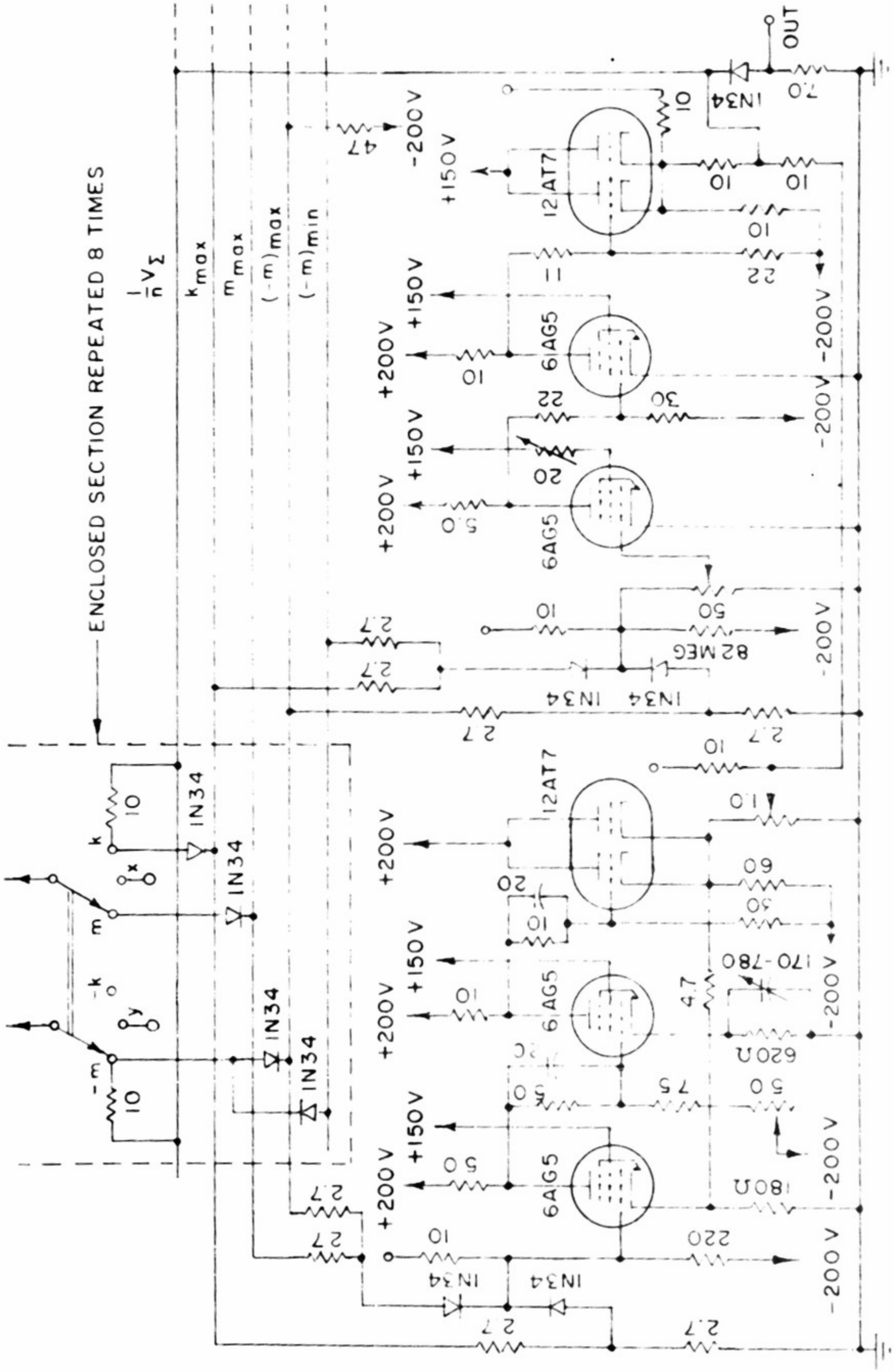


FIG. 4 THEORETICAL AND EXPERIMENTAL FREQUENCY RESPONSE OF MATCHED VIDEO FILTER.



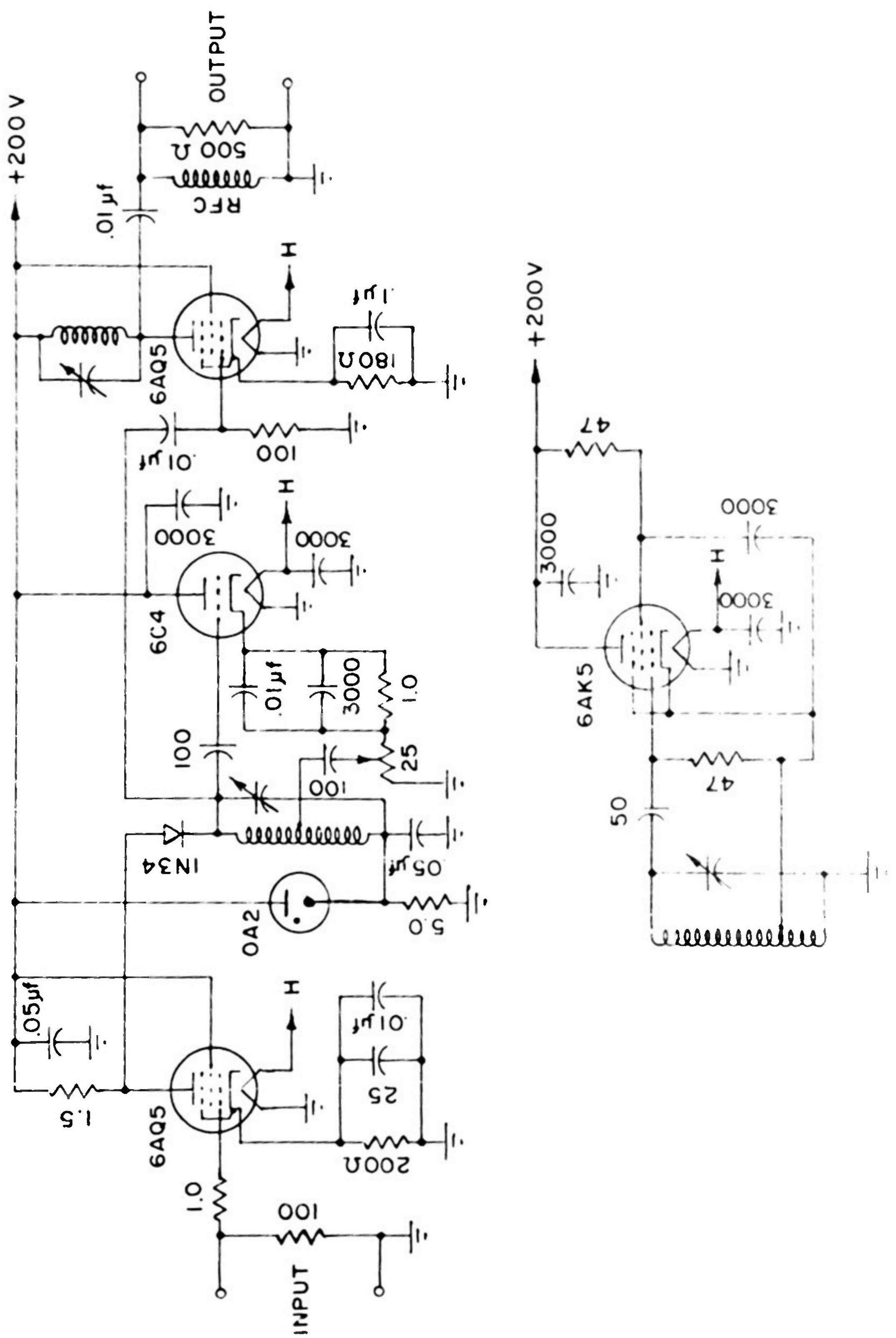
NOTE: ALL RESISTANCE VALUES IN KΩ AND ALL CAPACITANCE VALUES IN μμf UNLESS OTHERWISE SPECIFIED.

FIG. 5. REVISED SCHEMATIC OF PULSE-TRAIN CORRELATOR (FIRST HALF).



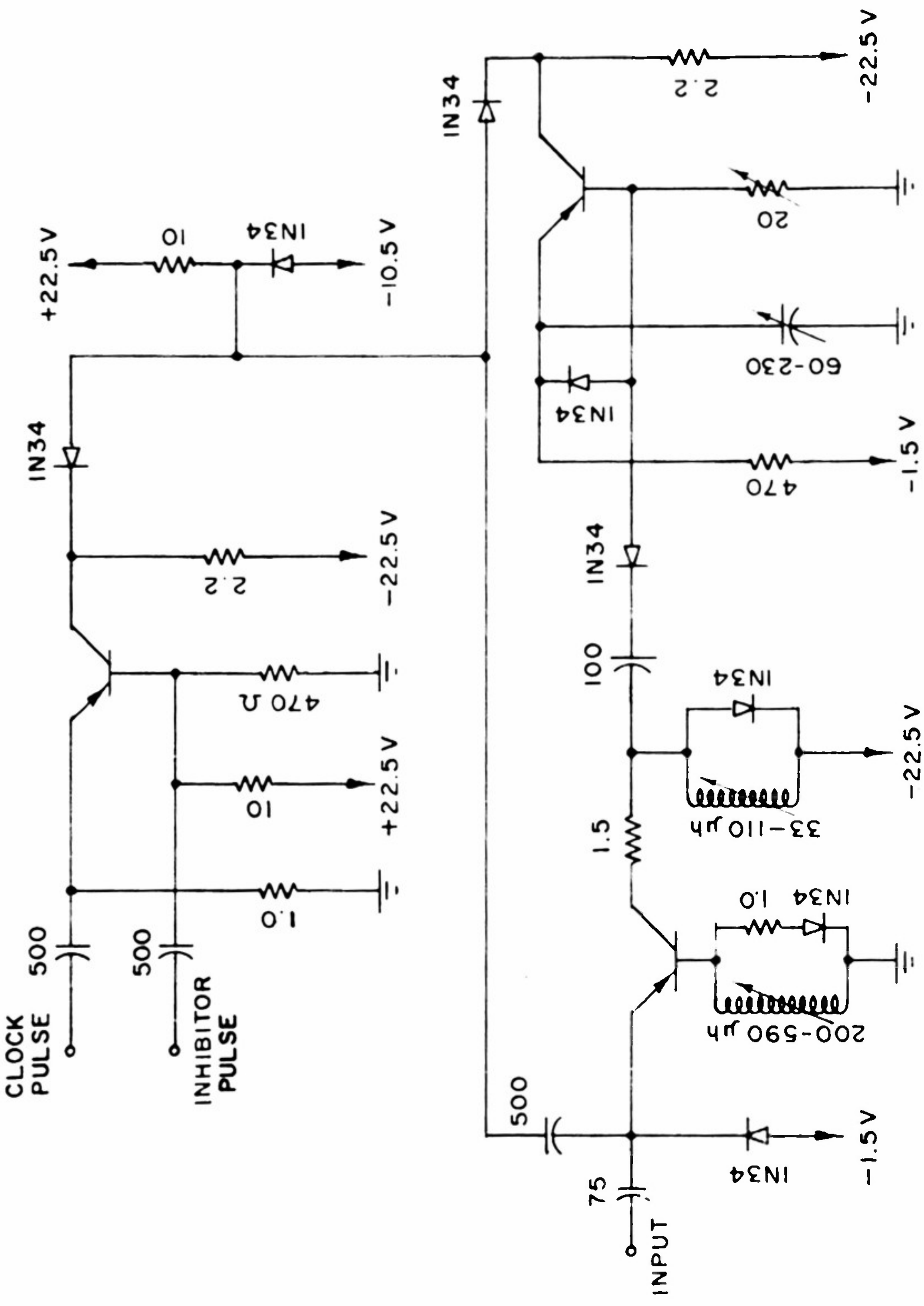
NOTE: ALL RESISTANCE VALUES IN KΩ AND ALL CAPACITANCE VALUES IN μμf UNLESS OTHERWISE SPECIFIED.

FIG. 6. REVISED SCHEMATIC OF PULSE-TRAIN CORRELATOR (SECOND HALF)



NOTE: ALL RESISTANCE VALUES IN KΩ AND ALL CAPACITANCE VALUES IN µµf UNLESS OTHERWISE SPECIFIED.

FIG. 7. WIRING DIAGRAM OF THE PULSE MODULATOR



NOTE: ALL RESISTANCE VALUES IN KΩ AND ALL CAPACITANCE VALUES IN μμf UNLESS OTHERWISE SPECIFIED

FIG. 8. DYNAMIC TRANSISTOR STORAGE CELL USING TWO MONOSTABLE CIRCUITS.

b. Derivations

Matched Filtering vs Conventional Band-Pass Filtering

A. Calculation of peak-signal power to noise-power ratio for matched i-f filter.

Previously it was shown* that a peak-signal to noise power ratio of

$$P_{S/N} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|E(\omega)|^2}{F(\omega)} d\omega$$

can be obtained when the filter characteristic is equal to

$$G(\omega) = K \frac{E(-\omega)\, \varepsilon^{-j\omega t'}}{F(\omega)} .$$

When the disturbance is white noise, its power spectral density is a constant $[F(\omega) = N_0]$ and the above expressions reduce to

$$P_{S/N} = \frac{1}{2\pi N_0} \int_{-\infty}^{\infty} |E(\omega)|^2 d\omega \qquad G(\omega) = \frac{K}{N_0} E(-\omega)\, \varepsilon^{-j\omega t'}$$

where $E(\omega) = \int_{-\infty}^{\infty} e(t)\, \varepsilon^{-j\omega t} dt,$

$e(t)$ = the signal,

t' = time at which filter output signal reaches its peak value,

and K = a constant.

Thus it can be seen that except for a constant time delay t', the matched filter should have a frequency response proportional to the conjugate of the transform of the signal e(t).

$$\text{Let} \quad e(t) = \begin{cases} V \cos\omega_0 t & \text{for } -\frac{\delta}{2} < t < \frac{\delta}{2} \\ 0 & \text{for all other time.} \end{cases}$$

$$\text{Then} \quad E(\omega) = \int_{-\delta/2}^{\delta/2} V \cos\omega_0 t\, \varepsilon^{-j\omega t} dt = 2V \int_0^{\delta/2} \cos\omega_0 t \cos\omega t\, dt$$

* Quarterly Progress Report No. 2, pp 6,7.

Thus $$E(\omega) = \frac{V\delta}{2}\left[\frac{\sin\frac{(\omega_0+\omega)\delta}{2}}{\frac{(\omega_0+\omega)\delta}{2}} + \frac{\sin\frac{(\omega_0-\omega)\delta}{2}}{\frac{(\omega_0-\omega)\delta}{2}}\right]$$

Since $E(\omega)$ is an even function, $E(\omega) = E(-\omega)$, and $G(\omega)$ should be made equal to

$$G(\omega) = \frac{K}{N_0} E(\omega)\, \epsilon^{-j\omega t'} .$$

Under these conditions

$$P_{S/N} = \frac{1}{2\pi N_0}\left(\frac{V\delta}{2}\right)^2 \int_{-\infty}^{\infty}\left[\frac{\sin\frac{(\omega_0+\omega)\delta}{2}}{\frac{(\omega_0+\omega)\delta}{2}} + \frac{\sin\frac{(\omega_0-\omega)\delta}{2}}{\frac{(\omega_0-\omega)\delta}{2}}\right]^2 d\omega$$

When $\frac{\omega_0\delta}{2} > 1$ (a condition usually met in a practical system) the integral of the cross-product term in the above integral is negligible and the expression may be approximated by

$$P_{S/N} \approx \left(\frac{1}{2\pi N_0}\right)\left(\frac{V\delta}{2}\right)^2 \int_{-\infty}^{\infty}\left\{\frac{\sin^2\frac{(\omega_0+\omega)\delta}{2}}{\left[\frac{(\omega_0+\omega)\delta}{2}\right]^2} + \frac{\sin^2\frac{(\omega_0-\omega)\delta}{2}}{\left[\frac{(\omega_0-\omega)\delta}{2}\right]^2}\right\} d\omega$$

$$P_{S/N}\Big]_{\frac{\omega_0\delta}{2} > 1} = \frac{V^2\delta}{2N_0} .$$

B. Calculation of peak-signal power to noise-power ratio for band-pass filter.

Let the signal e(t), immersed in a noise with a power spectral density of N_0, be applied to the input of a simple band-pass filter. In order to conform with the convention design of an i-f strip, the bandwidth, Δf, of the filter should be equal to twice the reciprocal of the pulse width. Consequently the damping factor of the filter will be

$$\frac{\Delta\omega}{2} = \pi\Delta f = \frac{1}{2RC} = \frac{2\pi}{\delta} .$$

Therefore, the initial transient may be assumed to be over by the end of the pulse (since $\epsilon^{-\frac{2\pi}{\delta}t} = \epsilon^{-2\pi} \approx 0$), and steady-state conditions will be in effect. Consequently the peak output signal will be equal to V, the peak input signal.

The average output-noise power is given by the expression

$$P_N = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 F(\omega)d\omega = \frac{N_o}{2\pi}\int_{-\infty}^{\infty}|H(\omega)|^2 d\omega$$

where $H(\omega)$ is the filter characteristic and is given by

$$H(\omega) = \frac{j\omega L}{(R - \omega^2 RLC) + j\omega L} .$$

Therefore

$$|H(\omega)|^2 = H(\omega)\ H(-\omega) = \frac{(\omega L)^2}{(R - \omega^2 RLC)^2 + (\omega L)^2}$$

Making the substitutions

$$\omega_o^2 = \frac{1}{LC} \quad \text{and} \quad \frac{1}{RC} = \frac{4\pi}{\delta} ,$$

we obtain

$$P_N = \frac{N_o}{2\pi}\int_{-\infty}^{\infty} \frac{\left(\frac{4\pi}{\delta}\right)^2}{4\sqrt{\omega_o^2 - \left(\frac{2\pi}{\delta}\right)^2}} \left[\frac{\omega}{\left(\omega - \sqrt{\omega_o^2 - \left(\frac{2\pi}{\delta}\right)^2}\right)^2 + \left(\frac{2\pi}{\delta}\right)^2} - \frac{\omega}{\left(\omega + \sqrt{\omega_o^2 - \left(\frac{2\pi}{\delta}\right)^2}\right)^2 + \left(\frac{2\pi}{\delta}\right)^2}\right] d\omega .$$

After integration the above expression reduces to

$$P_N = \frac{2\pi N_o}{\delta} .$$

Consequently the peak-signal power to noise-power ratio at the output of a band-pass filter becomes

$$P_{S/N} = \frac{V^2}{\frac{2\pi N_o}{\delta}} = \frac{V^2\delta}{2\pi N_o}$$

C. Calculation of improvement ratio.

The improvement achieved when a matched i-f filter is used in place of a conventional band-pass filter, can be expressed as the ratio of the result of part A to that of part B.

$$\text{Improvement ratio} = \frac{\dfrac{V^2\delta}{2N_o}}{\dfrac{V^2\delta}{2\pi N_o}} = \pi.$$

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